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Superconformal black hole quantum mechanics

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Abstract: In recent work, the superconformal quantum mechanics describing D0 branes in the $AdS_2 \times S^2 \times CY_3$ attractor geometry of a Calabi-Yau black hole with D4 brane charges p^A has been constructed and found to contain a large degeneracy of chiral primary bound states. In this paper it is shown that the asymptotic growth of chiral primaries for N D0 branes exactly matches the Bekenstein-Hawking area law for a black hole with D4 brane charge p^A and D0 brane charge N. This large degeneracy arises from D0 branes in lowest Landau levels which tile the $CY_3 \times S^2$ horizon. It is conjectured that such a multi-D0 brane CFT_1 is holographically dual to IIA string theory on $AdS_2 \times S^2 \times CY_3$.

Keywords: Black Holes in String Theory, D-branes, AdS-CFT and dS-CFT Correspondence.

Compactifications of type-II string theory on a Calabi-Yau space CY_3 contain extremal black hole solutions characterized by electric and magnetic charges (p^Λ, q_Λ). The near horizon region is an $AdS_2 \times S^2 \times CY_3$ attractor geometry [1, 2]. On general grounds [3], one expects the black hole attractor to be holographically dual to a conformally invariant quantum mechanics. However, despite enormous progress on AdS/CFT in higher dimensions, an understanding of AdS_2/CFT_1 has remained elusive. To date no complete examples are known.

In this paper we will pull together several observations and propose an AdS_2/CFT_1 correspondence, bringing to partial fruition the program initiated in [4−9]. In recent work [10] it has become clear that Calabi-Yau black holes are most naturally described in terms of fixed magnetic charges p^Λ and a weighted ensemble of electric charges q_Λ. This motivates the study of the quantum mechanical partition function of (electric) D0-branes in the background attractor geometry produced by (magnetic) D4 flux. The multi-D0 quantum mechanics is acted on by the (super) isometries of AdS_2 and is therefore automatically (super) conformally invariant [11, 12]. The quantum mechanics exhibits a rich spectrum of supersymmetric bound states, which can be described as D0-branes which pop out and in of the black hole horizon [13, 14]. Since they are not stationary with respect to asymptotic time, these bound states preserve only near-horizon (but no asymptotic) supersymmetries. Of particular interest are nonabelian N-D0 configurations corresponding to D2 branes which wrap the black hole horizon and carry N units of worldvolume magnetic flux. Such a D2-D0 brane effectively sees a magnetic flux (proportional to the D4 flux) on CY_3 and thereby acquires a large degeneracy of lowest Landau levels. The corresponding chiral primary states are counted and and found to exactly reproduce the leading order area-entropy formula for a D0-D4 black hole.

Based on this agreement at large-N in the BPS sector, we propose that the superconformal multi-D0 quantum mechanics is the holographic dual of string theory on the attractor geometry. Although we do not do so herein, this proposal might be sharpened and tested by examining subleading corrections to the entropy formula. This duality might also be extended to non-BPS excitations, taking due consideration of the subtleties in the AdS_2 scaling limit [15, 16].

Of course, the problem of microscopically computing the entropy of Calabi-Yau black holes has already been solved in the D0-D4 case [17, 18](see also [19, 20]). The new features here are the appearance of a dual CFT_1 (potentially relevant for non-BPS computations), and an interesting physical picture of the black hole microstates as D0 brane bound states localized near the horizon. We also hope that the method will generalize to Calabi-Yau black holes with a general set of charges, for which no microscopic derivation of the entropy is known.

To proceed, consider a black hole solution with D4 fluxes p^A, $A = 1, \ldots b_2$ and D0 flux q_0.[1] According to the attractor mechanism [1, 2], the radius R of the near horizon

[1]We will see later that the background q_0 drops out of the formula for chiral primaries. At this stage it is needed for a finite volume CY_3, but if we were to include the one-loop correction to the prepotential the volume may be finite in some cases even for $q_0 = 0$ [21].

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$AdS_2 \times S^2$ is equal to the graviphoton charge[2] Q

$$R = Q = \frac{g_s}{4\pi}\sqrt{\frac{D}{q_0}}\,, \qquad D \equiv D_{ABC}p^A p^B p^C\,, \tag{1}$$

with $6D_{ABC}$ the triple intersection numbers on CY_3. The Kahler class of CY_3 at the horizon is

$$J = \sqrt{\frac{q_0}{D}}p^A\omega_A, \tag{2}$$

where ω_A is an integral basis for $H^2(CY_3)$. There are also RR field strengths

$$F^{(4)} = \omega_{S^2}p^A\omega_A\,, \qquad F^{(2)} = q_0\omega_{AdS_2} \tag{3}$$

with the normalization $\int_{S^2}\omega_{S^2} = 1 = \int_{S^2\times CY_3} *\omega_{AdS_2}$.

We wish to compute a partition function for N D0 branes in the black hole attractor geometry (1)-(3). Consider

$$Z(q) \sim \mathrm{Tr}\left[\mathcal{O}q^N\right], \tag{4}$$

where the trace is over the multi-D0 Hilbert space and $\mathcal{O}$ is an appropriate operator insertion. In order to define and compute (4) we must choose a basis of states. A natural choice is eigenstates of the generator H of asymptotic black hole time, which corresponds to Poincaré time in the near horizon AdS_2. However for this choice (4) is generically ill-defined already for a single D0. As discussed in [8, 5, 7, 9], the problem arises from a divergent spectrum of low-energy states with arbitrarily large near-horizon redshifts.

To circumvent this problem, as in [12, 7, 8] we note that the D0-brane quantum mechanics has a (super)conformal structure inherited from the (super)isometries of the AdS_2 spacetime in which they move. This allows for the possibility [22] of using an alternate basis in (4): eigenstates of $L_0 = H + K$, where K is the generator of special conformal transformations. L_0 generates global (rather than Poincaré) time translations in AdS_2 [12]. It has a discrete spectrum and hence no problem with an IR divergent continuum. L_0 eigenstates are not static with respect to asymptotic black hole time, rather they pop out and in of the horizon, as depicted in figure 1. Here we propose that the black hole ground states should be identified with the chiral primaries of the near-horizon superconformal quantum mechanics. We emphasize that this identification is a proposal which should be tested and we do not have a derivation of its validity. Further discussion and motivation can be found in [7].

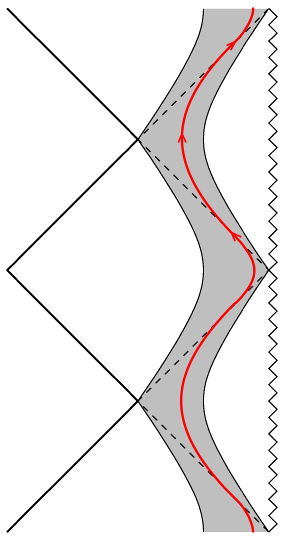

Figure 1: Penrose diagram for an extremal Calabi-Yau black hole. Each point represents an $S^2 \times CY_3$ except for the timelike singularity on the right. The near-horizon AdS_2 region is shaded, and the solid line within this region depicts the trajectory of an L_0 eigenstate popping in and out of the horizon.

[2] We will be working in the convention $2\pi\sqrt{\alpha'} = 1$, and the ten-dimensional Newton's constant $G_N = g_s^2/32\pi^2$. In the four-dimensional Planck units, we have $R = Q = (4Dq_0)^{\frac{1}{4}}$. In string units, $R = \frac{1}{2}g_s\sqrt{D\alpha'/q_0}$.

Supersymmetric chiral primary states were analyzed in detail in [13, 14], and come in several varieties. Nonabelian configurations of N D0 branes can, via the Myers effect [23], correspond to branes wrapping a non-trivial cycle Σ. In defining a black hole ground state partition function, we wish to fix the asymptotic charges, so we should not sum over configurations for which Σ is a nontrivial cycle in CY_3. This does not preclude configurations in which the Σ is the horizon S^2. Such a horizon-wrapped D2 does not lead to any asymptotic D2 charge in the full black hole geometry, and so such configurations should be summed over. In fact we see below that they are many of them at large-N.

The quantum mechanical hamiltonian and supercharges for various D0 configurations were constructed in detail in [13, 14]. Here we summarize the results for the case of interest, namely a D2 brane which wraps the horizon S^2 and carries N units of magnetic flux thereby inducing N units of D0 charge. Choosing the quantum mechanical time to be Poincaré time, these see an $R \times CY_3$ target space, with metric

$$ds^2 = T\left(4Qd\xi^2 + \frac{\xi^2}{Q}2g_{a\bar{b}}dz^a dz^{\bar{b}}\right), \tag{5}$$

where $g_{a\bar{b}}$ describes the Ricci-flat attractor geometry (2) on CY_3, T is the mass of a wrapped D2 with N units of flux

$$T = \frac{2\pi}{g_s}\sqrt{(4\pi Q^2)^2 + N^2} \tag{6}$$

and ξ is the spatial coordinate in which the AdS_2 metric is

$$ds^2 = Q^2\frac{-dt^2 + 4\xi^2 d\xi^2}{\xi^4}. \tag{7}$$

The D2 couples to the background RR flux $F^{(4)}$ of equation (3). Since this has two components tangent to the horizon, the D2 behaves like a point particle in CY_3 with a two-form magnetic field

$$F_{CY} = p^A\omega_A = \frac{4\pi Q}{g_s}J. \tag{8}$$

Altogether the bosonic part of the low-energy hamiltonian (generating $\frac{\partial}{\partial t}$) for a single D2 is

$$H = \frac{1}{8QT}P_\xi^2 + \frac{Q}{T\xi^2}(P_a - A_a)g^{a\bar{b}}(P_{\bar{b}} - A_{\bar{b}}) + \frac{32\pi^4 Q^5}{g_s^2 T\xi^2}, \tag{9}$$

where P_ξ, P_b are canonically conjugate to ξ, z^b. There are no angular fields on the S^2 in (9) because the D2 wraps the horizon. A here is the gauge field on CY_3 obeying

$$dA = F_{CY} = p^A\omega_A. \tag{10}$$

The last term in (9) is a potential which repels the D2 from the $\xi = 0$ AdS_2 boundary and pushes it towards the black hole horizon at $\xi = \infty$. It is the remnant of an imperfect cancellation between the brane mass and coupling to RR gauge fields. The precise form given here is deduced from supersymmetry in [14].

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The SU$(1,1|2)$ superisometries of the compactification must act on the Hilbert space of the D2 quantum mechanics. Hence the quantum mechanics has a complete set of superconformal generators of (a central extension of) SU$(1,1|2)$ which are given in [14]. The generator of special conformal transformations is

$$K = 2QT\xi^2 \,. \tag{11}$$

The generator of global AdS_2 time translations is

$$L_0 = H + K \,, \tag{12}$$

in which K acts as a potential barrier repelling the D2 from the black hole. Since H pushes the D2 toward the black hole, L_0 eigenstates are localized in the middle of the near-horizon AdS_2.

We are interested in the short multiplets of the superconformal algebra. These multiplets have chiral primary states annihilated by five of the eight superconformal charges, namely $G^{\alpha\beta}_{\frac{1}{2}}$ as well as

$$\begin{aligned} G^{++}_{-\frac{1}{2}} = (QT)^{-\frac{1}{2}} &\left[\frac{1}{2}\lambda^+ P_\xi - \frac{i}{\xi}(L^\eta)^+{}_\alpha\lambda^\alpha + \frac{i}{4\xi}\bar\lambda^+\lambda_\alpha\lambda^\alpha + \frac{i}{4\xi}\lambda^+\right] + \\ &+ \left(\frac{Q}{T}\right)^{\frac{1}{2}}\left[\frac{\sqrt{2}}{\xi}(\eta^a)^+(P_a - A_a) - \frac{8\pi^2 Q^2}{g_s}\frac{i}{\xi}\lambda^+\right] - (QT)^{\frac{1}{2}} 2i\xi\lambda^+ \,, \end{aligned} \tag{13}$$

where $\alpha, \beta = \pm$; $\lambda^\pm, \bar\lambda^\pm$ are four worldvolume Goldstinos, $(\eta^a)^\pm, (\bar\eta^{\bar a})^\pm$ are fermionic collective coordinates on the CY_3 and $(L^\eta)_{\alpha\beta} = g_{a\bar b}\eta^a_{(\alpha}\bar\eta^{\bar b}_{\beta)}$ are the SU(2) generators associated with the Lefschetz action on the CY_3.

The chiral primaries have an AdS_2 and CY_3 component of their wavefunction. We first consider the CY_3 component. The magnetic field divides the CY_3 into $D = \frac{1}{6}\int F_{CY} \wedge F_{CY} \wedge F_{CY}$ cells, corresponding the lowest Landau levels. Put another way, the D2 sees a non-commutative Calabi-Yau with non-commutativity parameter $\Theta^{-1} \sim F_{CY}$. There is roughly one chiral primary for each cell. To be more precise, the chiral primaries correspond to charged forms h obeying

$$\bar D h = \bar D^* h = 0 \tag{14}$$

where $iD_a \equiv P_a - A_a$. Such forms are in one to one correspondence with elements of $H^q(CY_3, \mathcal{L} \otimes \Omega^p)$, where $\mathcal{L}$ is the line bundle for which $c_1(\mathcal{L}) = [F_{CY}]$. For large c_1 and $q > 0$ $H^q(CY_3, \mathcal{L} \otimes \Omega^p)$ vanishes. $\dim H^0(CY_3, \mathcal{L} \otimes \Omega^p)$ can then be computed from the Riemann-Roch formula,

$$\begin{aligned} h_0 &\equiv \dim H^0(CY_3, \mathcal{L}) = \int \left(\frac{F_{CY}^3}{6} + \frac{c_2 \wedge F_{CY}}{12}\right) \\ &= D + \frac{1}{12} c_2 \cdot P \,, \\ h_1 &\equiv \dim H^0(CY_3, \mathcal{L} \otimes \Omega^1) = \int \left(\frac{F_{CY}^3}{2} - \frac{3c_2 \wedge F_{CY}}{4} + \frac{c_3}{2}\right) \end{aligned}$$

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$$
\begin{aligned}
&= 3D - \frac{3}{4}c_2 \cdot P - \frac{\chi}{2}\,, \\
h_2 \equiv \dim H^0(CY_3, \mathcal{L}\otimes\Omega^2) &= \int \left(\frac{F_{CY}^3}{2} - \frac{3c_2\wedge F_{CY}}{4} - \frac{c_3}{2}\right) \\
&= 3D - \frac{3}{4}c_2 \cdot P + \frac{\chi}{2}\,, \\
h_3 \equiv \dim H^0(CY_3, \mathcal{L}\otimes\Omega^3) &= \int \left(\frac{F_{CY}^3}{6} + \frac{c_2\wedge F_{CY}}{12}\right) \\
&= D + \frac{1}{12}c_2 \cdot P\,.
\end{aligned} \tag{15}
$$

Note that the background flux q_0 does not appear in these formulae, so in counting chiral primaries we can take $q_0 \to 0$. (15) may then be thought of as counting chiral primaries carrying total electric D0 charge N in the attractor background produced by the magnetic D4 charges p^A.

The picture of Calabi-Yau cells here is reminiscent of the one given in [19, 20], in which BPS states are described as D0 branes bound to the triple self-intersection sites of the D4 brane, except that here the explicit D4 branes are replaced by D4 fluxes. What has happened is that in the brane-geometry transition the self-intersection sites have dissolved into lowest Landau levels for the D0 branes or, equivalently, the cells of the noncommutative Calabi-Yau. There may also ultimately be a relation to the crystal atoms of [24].

Now we turn to the AdS_2 component of the wavefunction. Given a form of the type (14) one can act on it with the Lefschetz raising operator $(L^\eta)^{++}$ and obtain a new form $\tilde{h}$ that satisfies

$$
D\tilde{h} = D^*\tilde{h} = 0\,. \tag{16}
$$

When the CY_3 component corresponds to a form of the type (16), $G^{++}_{\pm\frac{1}{2}}$ reduce to

$$
G^{++}_{\pm\frac{1}{2}} = \frac{\lambda^{++}}{\sqrt{QT}}\left[\frac{1}{2}P_\xi - \frac{i}{\xi}(L^\eta)^{+-} + \frac{i}{2\xi}\lambda^{+-}\lambda^{-+} + \left(\frac{1}{4} - \frac{8\pi^2}{g_s}Q^3\right)\frac{i}{\xi} \mp 2iQT\xi\right]. \tag{17}
$$

These annihilate the state $|0\rangle$ defined by $\lambda^{++}|0\rangle = 0$. When acting on $|0\rangle$, $G^{--}_{\frac{1}{2}}$ reduces to

$$
\begin{aligned}
G^{--}_{\frac{1}{2}} &= \frac{\lambda^{--}}{\sqrt{QT}}\left[\frac{1}{2}P_\xi + \frac{i}{\xi}(L^\eta)^{-+} - \frac{i}{2\xi}\lambda^{+-}\lambda^{-+} - (\frac{1}{4} - \frac{8\pi^2}{g_s}Q^3)\frac{i}{\xi} - 2iQT\xi\right] - \\
&\quad - \frac{i}{\xi\sqrt{QT}}(L^\eta)^{--}\lambda^{+-}\,.
\end{aligned} \tag{18}
$$

This is solved by the AdS_2 wave function

$$
|\psi\rangle = \xi^{2L_3^\eta + 16\pi^2 Q^3/g_s + \frac{1}{2}} \exp(-2QT\xi^2)|0\rangle \otimes \tilde{h} \tag{19}
$$

where the fermionic part of the state obeys

$$
\lambda^{++}|0\rangle = \lambda^{+-}|0\rangle = 0\,. \tag{20}
$$

(19) is normalizable for positive Q. It follows from the superconformal algebra [14] that $G^{+-}_{\frac{1}{2}}|\psi\rangle = 0$, and $|\chi\rangle = G^{-+}_{\frac{1}{2}}|\psi\rangle \neq 0$. Now $|\chi\rangle$ is a chiral primary state.

We now count the multiparticle chiral primaries. First we consider all possible ways that the D0s can form D2s.[3] We partition the N D0 branes into k clusters of n_i D0 branes each such that

$$\sum_{i=1}^{k} n_i = N\,. \tag{21}$$

Each cluster then forms a wrapped D2 brane with n_i units of magnetic flux. Each of the k such D2 branes can then sit in one of the $h_0 + h_1 + h_2 + h_3$ chiral primary states. Noting the chiral primary states associated to h_p have quantum mechanical fermion number p, the counting of such configurations is the same as the counting of states of a 1+1 CFT with $h_0 + h_2$ bosons, $h_1 + h_3$ fermions and total left-moving momentum N. The chiral primary generating function is then

$$Z = \mathrm{Tr}\, q^N = \prod_n \frac{(1+q^n)^{h_1+h_3}}{(1-q^n)^{h_0+h_2}}\,, \tag{22}$$

where the trace is over chiral primaries. Using the values (15) for h_r and the well known asymptotics of (22) we find an asymptotic formula for entropy as a function of N

$$S = 2\pi\sqrt{D_{ABC}p^A p^B p^C N}\,, \tag{23}$$

in precise agreement with the macroscopic area law for a black hole with D4 charge p^A and D0 charge N. This is evidence for the proposal that near horizon chiral primaries count black hole ground states.

Let us now briefly comment on what would be needed to promote this to a full-blown AdS_2/CFT_1 correspondence. First there are various $\frac{1}{N}$ corrections to the BPS degeneracies. These come for example from the subleading terms in (15) as well as configurations with D0 charge but no wrapped D2s. The superconformal quantum mechanics also has computable non-BPS excitations. This might be identified with near-extremal black hole microstates. The total D0 charge of the system is $N+q_0$, the sum of the number of explicit D0 branes plus the background flux. In the non-BPS sector the quantum mechanics has q_0 dependence, so one must understand the $q_0 \to 0$ limit in which the target CY_3 scales to zero size, among other issues.

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